Strict inequalities between bond percolation thresholds of Archimedean lattices.

John C. Wierman, Johns Hopkins University

A percolation model is an infinite random graph model for phase transitions and critical phenomena. The percolation threshold corresponds to a phase transition point, such as a melting or freezing temperature. The exact value of the percolation threshold is known for only a few two-dimensional percolation models.

An Archimedean lattice is an infinite vertex-transitive graph that tiles the plane with regular polygons. There are eleven Archimedean lattices, which are of interest to physicists studying percolation models. However, most information about their percolation thresholds has been obtained by simulations.

Recent computational advances in the substitution method produce new rigorous bounds for the bond percolation thresholds of Archimedean lattices, which prove several new strict inequalities. The substitution method is based on stochastic ordering of probability measures on partition lattices, and translates an exact value or bound for the percolation threshold of one lattice into a bound for the percolation threshold of another lattice.

Keywords: bond percolation, percolation threshold, Archimedean lattice, stochastic ordering, partition lattice.