The Hall spectrum of a graph can have non-consecutive zeros

Victor Larsen, Jennifer Vandenbussche, Erik E. Westlund*, Kennesaw State University

Hall’s condition is necessary (though not sufficient) for a graph to admit a precoloring extension. A graph is Hall $k$-extendible if every $k$-precoloring satisfying Hall’s condition is extendible to a $k$-coloring. It is known that every graph $G$ is Hall $k$-extendible for all $k \geq \Delta(G)$. The Hall spectrum of a graph is a binary vector whose entries indicate the values of $k$ for which the graph is Hall $k$-extendible. We discuss recent work on Hall spectrums and show that it is possible for the Hall spectrum of a graph to have non-consecutive zeros, thereby answering a question of Bobga, Goldwasser, Hilton, and Johnson posed in 2011. We also discuss new results on extending Hall precolorings with extra colors.

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