Rainbow-Cycle-Forbidding Edge Colorings of Complete Multipartite Graphs, with the Maximum Possible Number of Colors Appearing

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It is well known that if the edges of a finite simple graph on $n$ vertices are colored so that no cycle is rainbow, then no more than $n - 1$ colors can appear on the edges. In previous work it has been shown that the essentially different rainbow-cycle-forbidding edge colorings of $K_n$ with $n - 1$ colors appearing are in 1-1 correspondence with (can be encoded by) the (isomorphism classes of) full binary trees with $n$ leafs. In the encoding, the natural Huffman labeling of each tree arising from the assignment of 1 to each leaf plays a role. Very recently it has been shown that a similar encoding holds for rainbow-cycle-forbidding edge colorings of $K_{a,b}$ with $a + b - 1$ colors appearing. In this case the binary trees are given Huffman labelings arising from certain assignments of (0,1) or (1,0) to the leafs. (Sibling leafs are not allowed to be assigned the same label.) In this paper we prove the analogous result for complete $r$-partite graphs, for $r > 2$.

Keywords: rainbow-cycle-forbidding, edge coloring