Degree complete graphs

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Let $G = (V, E)$ be a graph with an ordered vertex set $V = \{1, \ldots, n\}$. We call a vector of nonnegative integers $S = (s_1, \ldots, s_n)$ a degree vector of $G$ if there is an orientation $D$ of $G$ such that $s_i = d^+_D(i)$ for all $i \in V$. It is known that every degree vector satisfies

$$S_G^l \preceq S \preceq S_G^r, \quad \sum_{i=1}^n s_i = |E| \quad \text{and} \quad 0 \leq s_i \leq d_G(i) \quad \text{for all} \quad i = 1, \ldots, n, \quad (1)$$

where $S_G^l$ ($S_G^r$) is the minimal (maximal) degree vector of $G$ with respect to the domination order. The graph $G$ is called degree complete if every vector $s$ satisfying condition (1) is a degree vector of $G$. In 2006 Qian characterized degree complete graphs with ordered vertex sets.

Suppose we are given a graph $G$ without an ordered vertex set Qian asked how to decide whether there is an ordering of the vertices of $G$ that yields a degree complete graph. Answering this problem we give two characterizations of the class of graphs which have such a degree complete vertex set ordering. Moreover we state a polynomial procedure to find a desired ordering of the vertices of $G$ if it exists.

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