Bounds for the Sum Choice Number

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Let $G = (V(G), E(G))$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$, and for every vertex $v \in V(G)$ let $L(v)$ be a set (list) of available colors. The graph $G$ is called $L$-colorable if there is a proper coloring $\phi$ of the vertices with $\phi(v) \in L(v)$ for all $v \in V(G)$. A function $f$ from the vertex set $V(G)$ of $G$ to the positive integers is called a choice function of $G$ and $G$ is said to be $f$-list colorable if $G$ is $L$-colorable for every list assignment $L$ with $|L(v)| = f(v)$ for all $v \in V(G)$. Set $\text{size}(f) = \sum_{v \in V(G)} f(v)$ and define the sum choice number $\chi_{sc}(G)$ as minimum of $\text{size}(f)$ over all choice functions $f$ of $G$.

It is easy to see that $\chi_{sc}(G) \leq |V(G)| + |E(G)|$ for every graph $G$ and that there is a greedy coloring of the vertices of $G$ for the corresponding choice function $f$ and every list assignment $L$ with $|L(v)| = f(v)$ for all $v \in V(G)$.

Obviously, if $\chi_{sc}(G) \leq k$ and $H$ is a subgraph of $G$, then $\chi_{sc}(H) \leq k$. Therefore, this property is a hereditary graph property. This implies $\chi_{sc}(G) \geq 2|V(G)| - 1$ for a connected graph $G$ since $\chi_{sc}(T) = 2|V(G)| - 1$ for a spanning tree $T$ of $G$.

In this talk we will improve the above mentioned upper and lower bounds for the sum choice number. We will present several general lower and upper bounds on $\chi_{sc}(G)$ in terms of subgraphs of $G$.

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