Embedding Complete Multipartite Graphs into Smallest Dimension

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For a finite, simple graph $G$, define $G$ to be of dimension $d$ if $d$ is the minimum value such that $G$ can be drawn with vertices being points of $\mathbb{R}^d$ where adjacent vertices are necessarily placed a unit distance apart. We determine the dimension of all complete multipartite graphs. Letting $G$ be a complete multipartite graph with $n$ parts, $m$ of which have size one or two, our main result is that $G$ is of dimension $2n - m - 1$ when all parts or all but one part of $G$ has size one, and $G$ is of dimension $2n - m$ otherwise.

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