Up-Down Ternary Strings

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Let $\Sigma$ be the alphabet $\{0, 1, 2\}$, ordered as $0 < 1 < 2$. For $n \geq 2$, an element $x_1 x_2 \ldots x_n \in \Sigma^n$ is called an up-down (ternary) string of length $n$ when $x_1 < x_2$, $x_2 > x_3$, $\ldots$, and $x_{n-1} < x_n$, for $n$ even, while $x_{n-1} > x_n$, for $n$ odd. If $n = 4$, for instance, there are eight such ternary strings, namely

$$0101, 0102, 0212, 0201, 0202, 1212, 1201, 1202.$$ 

In general, the number of such strings of length $n$ is given by $F_{n+2}$, the $(n + 2)$nd Fibonacci number. Among the statistics for these $F_{n+2}$ strings of length $n$, we determine

(1) the number of occurrences of each of 0, 1, and 2; (2) the sum of these strings considered as base 3 integers; and, (3) the number of inversions that occur.

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