A family of 2-crossing-critical graphs on the projective plane

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A graph $G$ is said to be 2-crossing-critical if it has crossing number at least two and every proper subgraph of $G$ has crossing number less than two. Bokal, Oporowski, Richter, and Salazar recently determined all the 3-connected 2-crossing-critical graphs containing a subdivision of the Möbius Ladder $V_{10}$. These graphs are members of a family generated by joining certain tiles in sequence. We show a closely related family of tile joins that are 2-crossing-critical on the real projective plane. Analogous to the plane case, these graphs have projective crossing number at least two and each proper subgraph has projective crossing number less than two. We also discuss ongoing work toward extending this family to all non-orientable surfaces.

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