Enumerating Hamiltonian Paths in Cayley Digraphs on the Semidirect Product of Cyclic Groups

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Let $\epsilon_1, \epsilon_2 \in \{-1, 1\}$. Let $m$ and $n$ be odd positive integers such that $n = \epsilon_2 \pmod{4}$. Let $G$ be the semidirect product of cyclic groups of order $8m$ and $2n$ given by $G = \mathbb{Z}_{8m} \rtimes \mathbb{Z}_{2n} = \langle x, y : x^{8m} = 1, y^{2n} = 1, \text{ and } yxy^{-1} = x^{2m\epsilon_1-1} \rangle$. The Cayley digraph on $G$ with generating set $\{x, y\}$, denoted by Cay$(G : x, y)$, is the digraph with vertex set $G$ and arc set consisting of arcs from $g$ to $gs$ whenever $g \in G$ and $s \in \{x, y\}$. We determine a formula for the number of hamiltonian paths in Cay$(G : x, y)$ (with initial vertex 1) that depends on the parameters $m, n, \epsilon_1$ and $\epsilon_2$. In addition, we determine all hamiltonian paths in Cay$(G : x, y)$ (with initial vertex 1).

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